QUANTUM MACHINE LEARNING

Quantum Annealer for Network Flow Minimization in InSAR Images

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Quantum Annealer for Earth observation

Our objective is to employ and investigate a D-Wave quantum annealer to real-world problems in Earth observation.

- Why we have to employ and investigate a quantum annealer (in general)?

1. A quantum annealer is made a huge progress in terms of scaling (qubits) and connectivity of qubits;
2. Such a device is well-suited for a particular class of hard-problems (e.g., Boltzmann Sampling). Otherwise, this particular class of problems is ill-suited on a conventional computer;
3. Well-suited, real-world problems for a D-Wave quantum annealer to be found is *an ongoing challenge*;

Therefore, we are exploring real-world problems in Earth observation which are well-suited for finding their solutions on a quantum annealer over a conventional computer. A quantum annealer might prove relevant even if we are not intended to exhibit its advantages over a conventional computer.

- We attempt to answer following questions in this work:
    - What is a D-Wave quantum annealer and How does it work?
    - Which problem in Earth observation?
    - How to implement our chosen problem in a D-Wave quantum annealer?
    - What challenges may we encounter for programming a D-Wave quantum annealer?

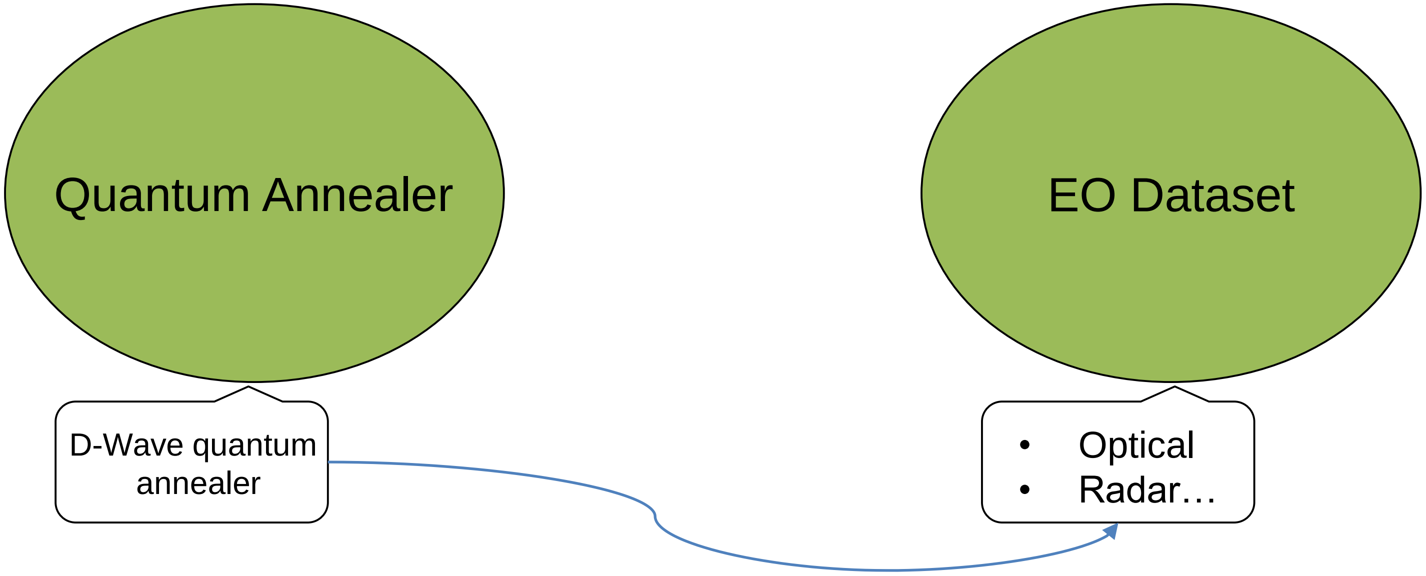
Main Contributions

- **Given:**
  - **Chosen Problem:** Residue Connection problem; namely Network Flow Minimization (NFM)
  - **D-Wave Quantum Annealer:** Quadratic Unconstrained Binary Optimization (QUBO) problem

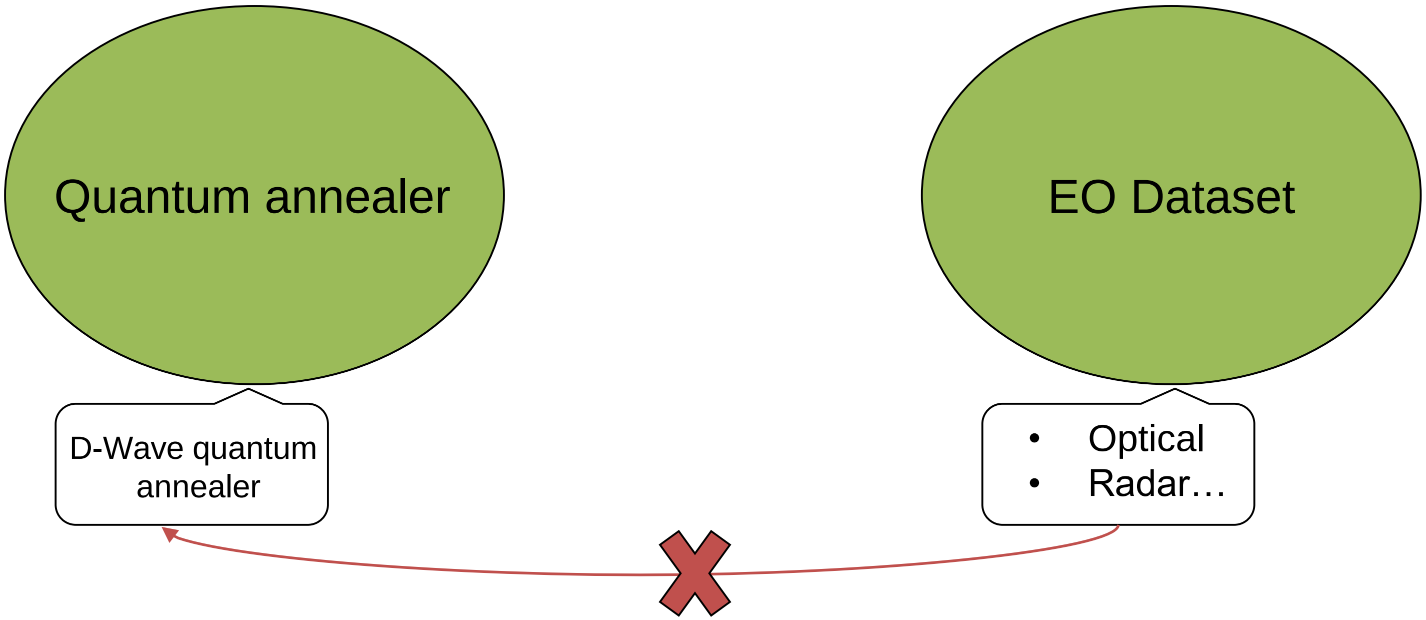
- **Our Contributions:**
  - NFM-to-QUBO formulation
  - Tuning parameters/programming of a D-Wave Quantum Annealer
  - Classical annealer VS Quantum annealer
  - To gain insight on a D-Wave quantum annealer

- **Conclusion**: A D-Wave quantum annealer outperforms a classical annealer.

Quantum annealer for Earth observation

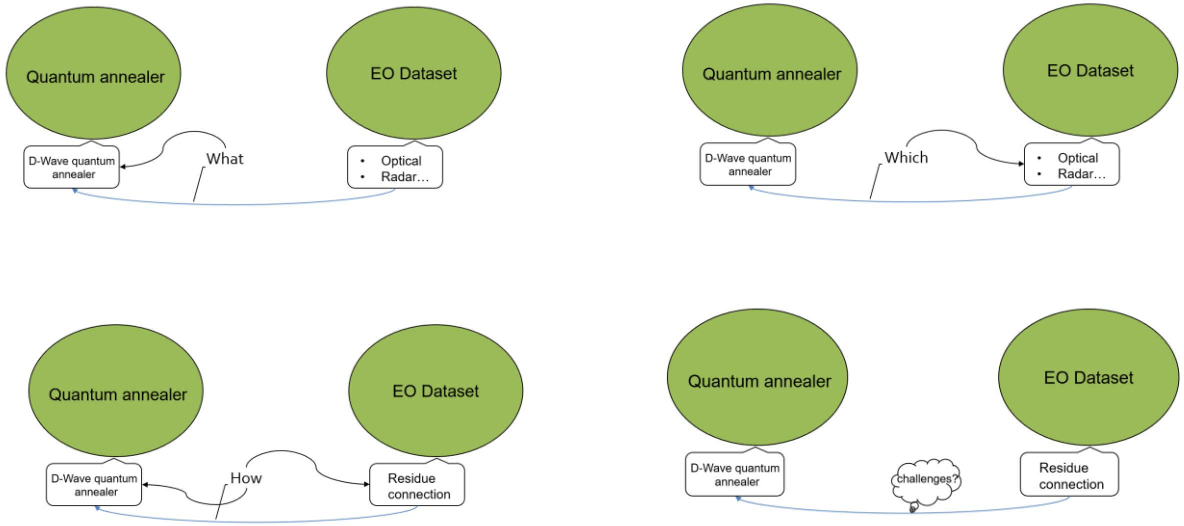


Quantum annealer for Earth observation



Quantum annealer for Earth observation

- What is a D-Wave quantum annealer and How does it work?
- Which problem in Earth observation?
- How to implement our chosen problem in a D-Wave quantum annealer?
- What challenges may we encounter for using a D-Wave quantum annealer?



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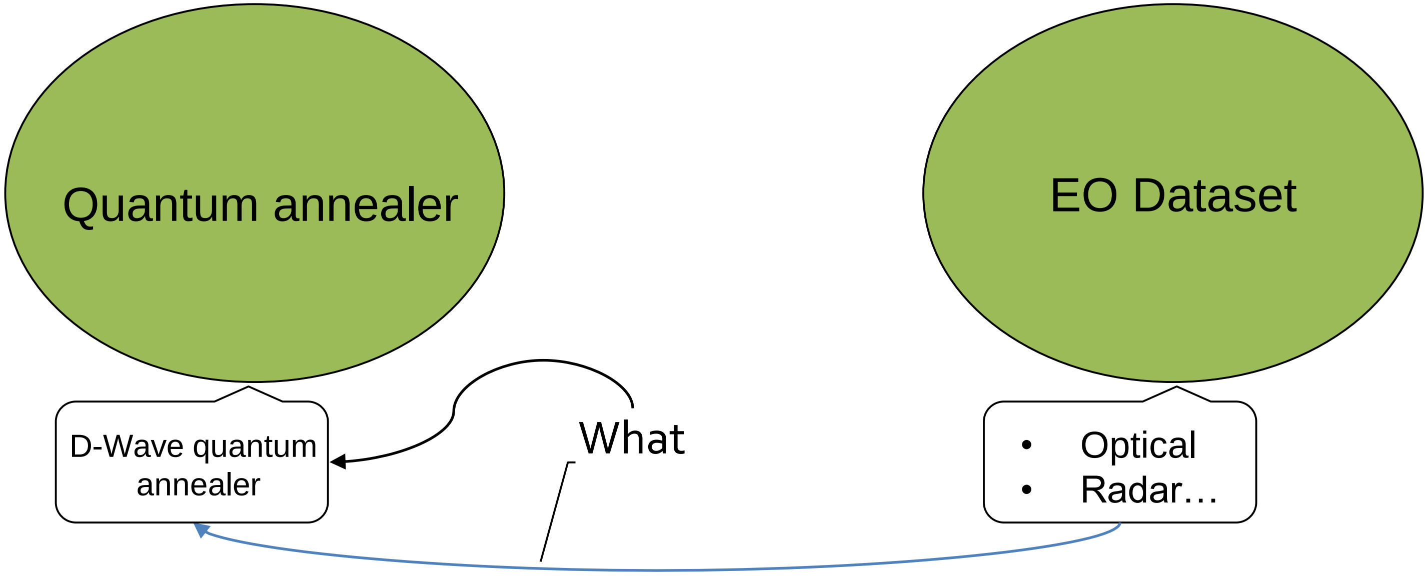
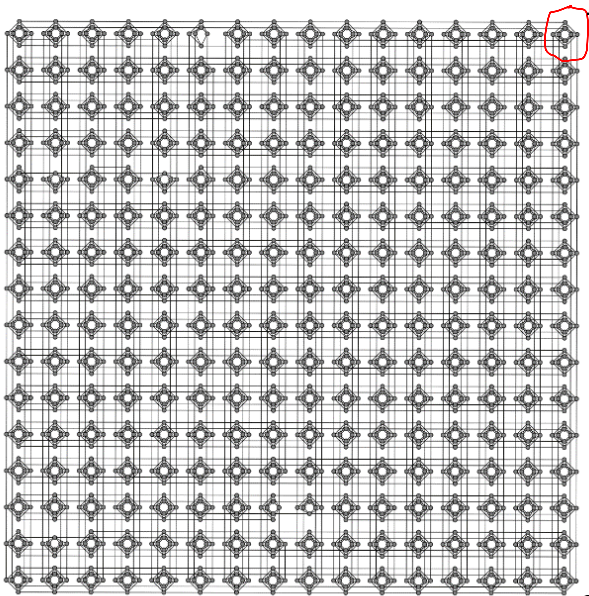
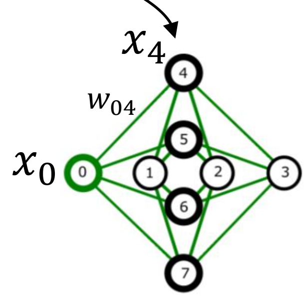
Quantum annealer for Earth observation



What is a D-Wave quantum annealer and How does it work?

Topology (Chimera or Pegasus):
Constrained connectivity among qubits





$x_i \in \{-1, +1\}$

D-Wave quantum annealer:

- I. Energy-Based method (Ising or QUBO model)
- II. Annealing
- III. Vertices can be in states ups or downs.

Not heuristic and a promise to reach a global minimum.

$$x_i = 1 - 2b_i$$

$$E(x) = -\sum_{ij} w_{ij} x_i x_j - \sum_i c_i\, x_i\,, \mathrm{x_i} \in \{-1, +1\}$$

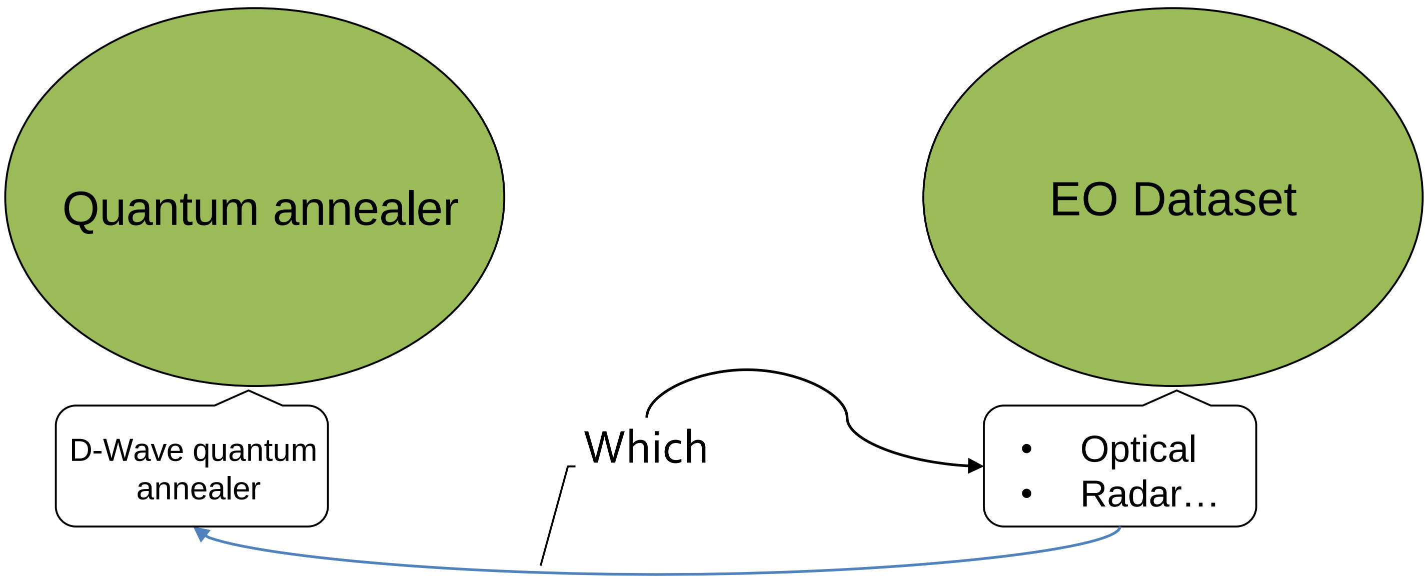
Ising

$$E(b) = \sum_{ij} q_{ij} b_i b_j + \sum_i q_i\, b_i\,, b_i \in \{0, +1\}$$

QUBO

Arxiv:2101.08448

Which problem in Earth observation?



Residue connection in InSAR

1. D-Wave quantum annealer:
   I. Energy-Based method (Ising or QUBO model)
   II. Annealing
   III. Vertices can be in states ups or downs.

Not heuristic and a promise to reach a global minimum.

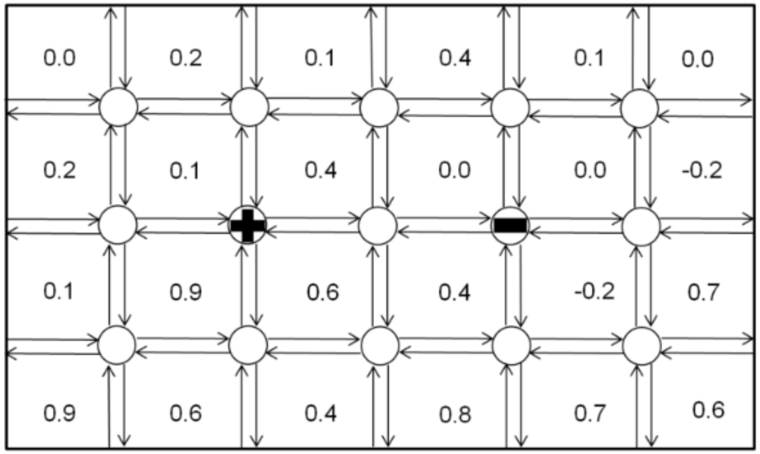
2. (Mixed) Integer Problems (MIP):
   I. Integer or Continuos
   II. Non Convex
   III. NP-Hard

Heuristic methods are employed; the method is not guaranteed to converge into a global minimum (DL).

QUBO -> MIP

Therefore, (researchers assume) a D-Wave quantum annealer is well-suited to find a solution of Quadratic Unconstrained Binary Optimization (QUBO) problems.

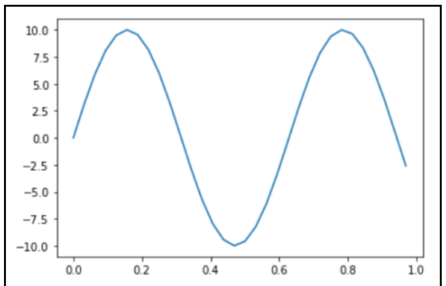
Integer Optimization (MIP): NFM



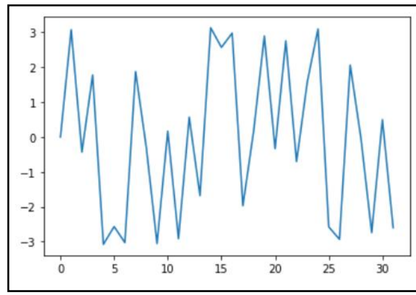
M. Costantini, "A novel phase unwrapping method based on network programming, "IEEE Transactionson Geoscience and Remote Sensing, vol. 36, pp. 813-821, 1998.

Quantum annealer for residue connection

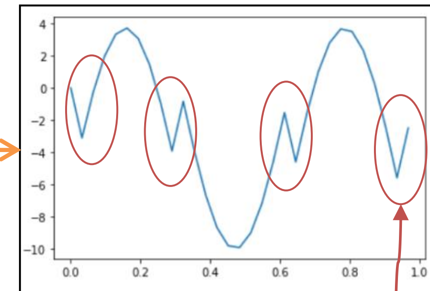
1D Signal



1D Wrapped Signal



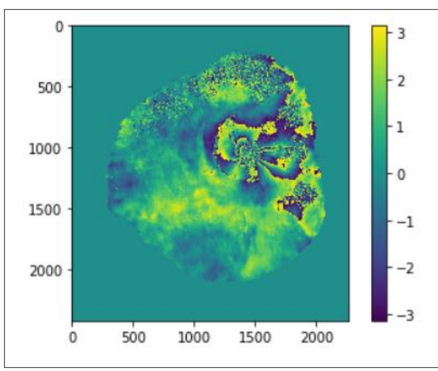
1D Unwrapped Signal



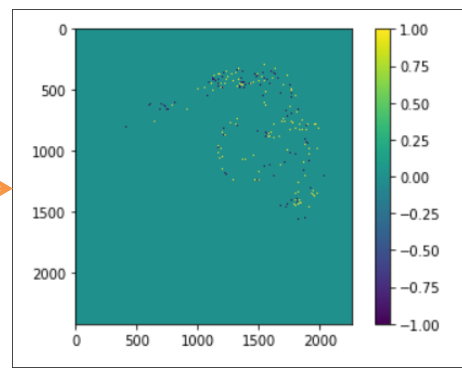
Due to noise
(Residues)
in a received signal.

Not the same result after unwrapping the wrapped signal.

2D Signal

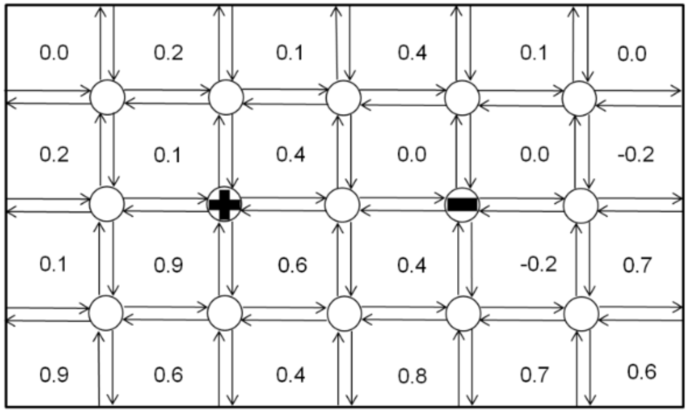


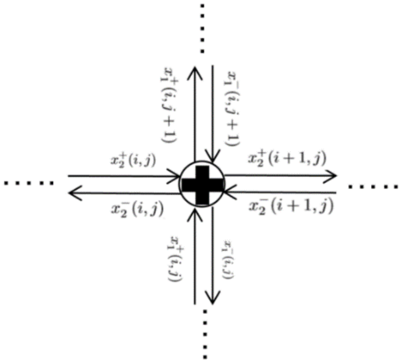
Residue in 2D Signal



Residue can be visualized easily on 2D signal.

Residue connection: Network Flow Minimization





$$\min_{\{x_1^+,x_1^-,x_2^+,x_2^-\}} \sum_{i=0}^{N-2}\sum_{j=0}^{M-1} c_1(i,j)[x_1^+(i,j)+x_1^-(i,j)] + \sum_{i=0}^{N-1}\sum_{j=0}^{M-2} c_2(i,j)[x_2^+(i,j)+x_2^-(i,j)],$$

$$\underbrace{\begin{aligned}&x_1^+(i,j+1)-x_1^-(i,j+1)-x_1^+(i,j)+x_1^-(i,j)\\&-x_2^+(i+1,j)+x_2^-(i+1,j)+x_2^+(i,j)-x_2^-(i,j)\end{aligned}}_{A\vec{x}} = \underbrace{-\frac{1}{2\pi}(\psi_1(i,j+1)-\psi_1(i,j)-\psi_2(i+1,j)+\psi_2(i,j))}_{\vec{b}},$$

M. Costantini, “A novel phase unwrapping method based on network programming, ”IEEE Transactionson Geoscience and Remote Sensing, vol. 36, pp. 813-821, 1998.

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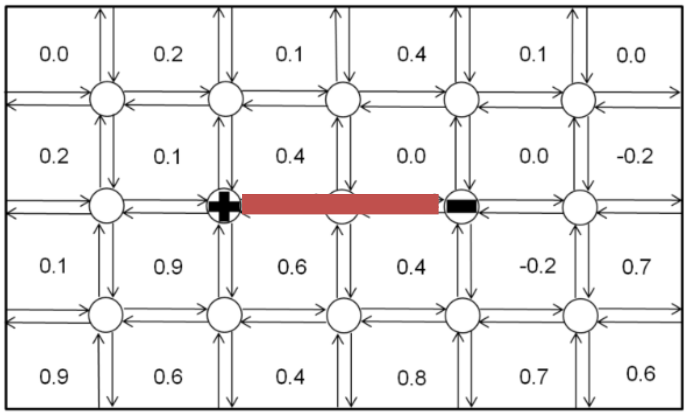
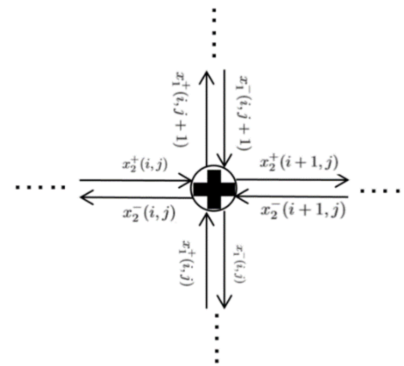
Residue connection: Network Flow Minimization



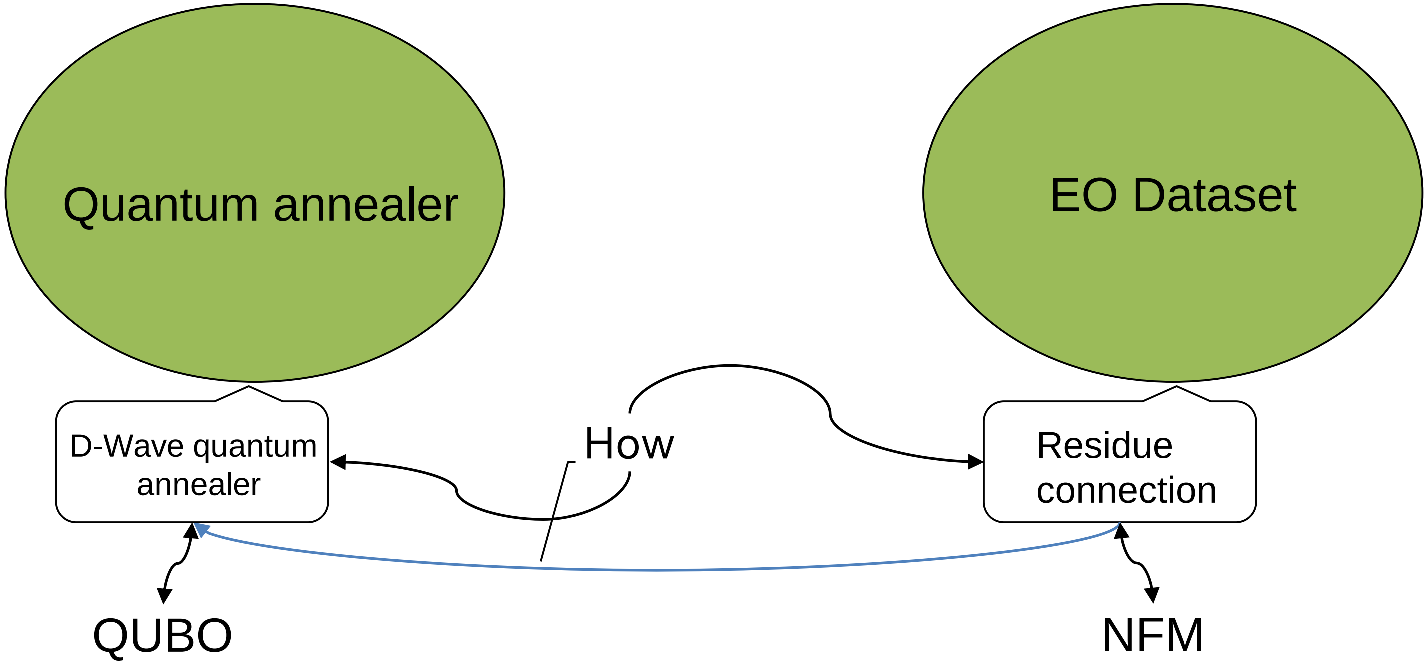


$$\min_{\{x_1^+,x_1^-,x_2^+,x_2^-\}} \sum_{i=0}^{N-2}\sum_{j=0}^{M-1} c_1(i,j)[x_1^+(i,j)+x_1^-(i,j)] + \sum_{i=0}^{N-1}\sum_{j=0}^{M-2} c_2(i,j)[x_2^+(i,j)+x_2^-(i,j)],$$

$$\underbrace{\begin{aligned}&x_1^+(i,j+1)-x_1^-(i,j+1)-x_1^+(i,j)+x_1^-(i,j)\\&-x_2^+(i+1,j)+x_2^-(i+1,j)+x_2^+(i,j)-x_2^-(i,j)\end{aligned}}_{A\vec{x}}$$

$$= \underbrace{-\frac{1}{2\pi}(\psi_1(i,j+1)-\psi_1(i,j)-\psi_2(i+1,j)+\psi_2(i,j)),}_{\vec{b}}$$

Quantum annealer & residue connection



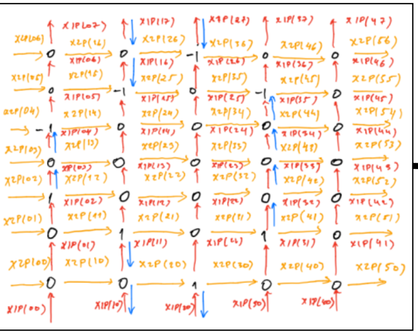
How to implement our chosen problem in a D-Wave quantum annealer?

Residues must be avoided or connected; So,

1. A problem of the residue connection is formulated as a Network Flow Minimization (NFM) minimization problem (Fig. 1);
2. A NFM minimization problem must be written as a QUBO.
3. A D-Wave quantum annealer solves our QUBO problem (e.g., finding a global solution).

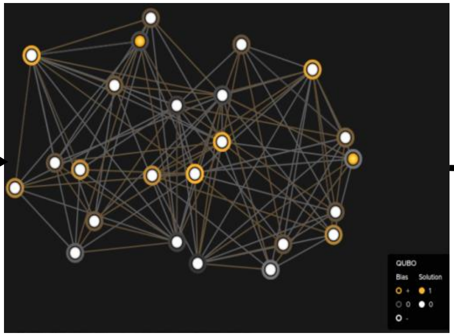
Therefore, we formulate a NFM problem as a QUBO problem and solve it on a D-Wave quantum annealer. A D-Wave quantum annealer might prove relevant even if we are not intended to exhibit the computational advantage over a conventional method, but we aimed to formulate a QUBO and embed it into a topology of a D-Wave quantum annealer.

Fig. 1: Residue in 2D Signal



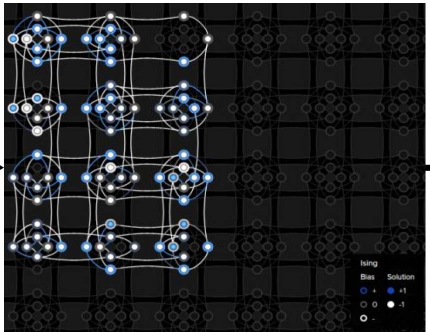
NFM

Residue in 2D Signal

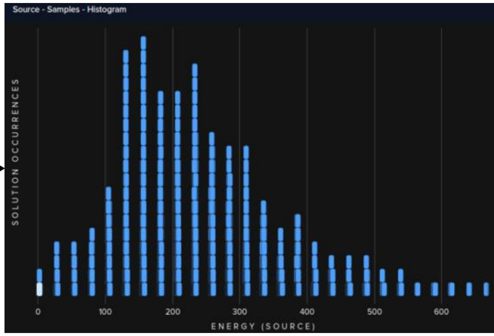


QUBO

Embedding in D-Wave quantum annealer



Solution in D-Wave quantum annealer



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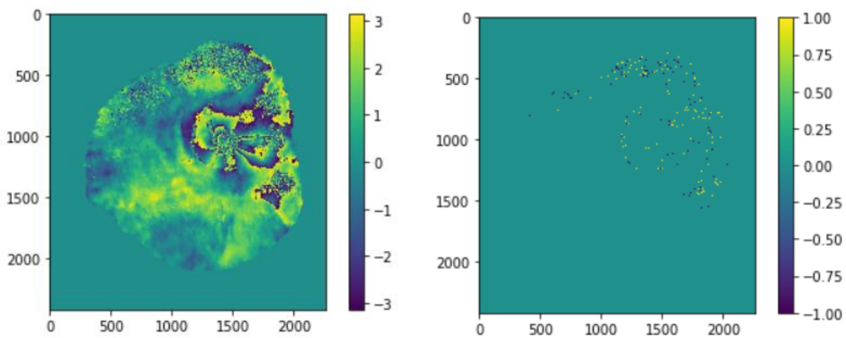
A dataset: Cape Verde Volcano

Our dataset is Cape Verde Volcano (Fig. 2a), and we extracted residues with a size of 3x3 and 7x5 elements (Fig. 2b) such that $x_1^{\pm} \in \{0, +1\}$, $x_2^{\pm} \in \{0, +1\}$ of a NFM formulation:

$$\min_{\{x_1^+, x_1^-, x_2^+, x_2^-\}} \sum_{i=0}^{N-2} \sum_{j=0}^{M-1} c_1(i,j)[x_1^+(i,j) + x_1^-(i,j)] + \sum_{i=0}^{N-1} \sum_{j=0}^{M-2} c_2(i,j)[x_2^+(i,j) + x_2^-(i,j)],$$



Cape Verde Volcano

Residues

(a)

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 \end{bmatrix}.$$

(b)

https://www.esa.int/

Fig. 2

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NFM-to-QUBO formulation

A NFM formulation:

$$\min_{\{x_1^+,x_1^-,x_2^+,x_2^-\}} \sum_{i=0}^{N-2}\sum_{j=0}^{M-1} c_1(i,j)[x_1^+(i,j)+x_1^-(i,j)]$$
$$+\sum_{i=0}^{N-1}\sum_{j=0}^{M-2} c_2(i,j)[x_2^+(i,j)+x_2^-(i,j)],$$

$$\underbrace{[x_1^+(i,j+1)-x_1^-(i,j+1)]+[-x_1^+(i,j)+x_1^-(i,j)]}_{x_1}$$
$$+\underbrace{[-x_2^+(i+1,j)+x_2^-(i+1,j)]+[x_2^+(i,j)-x_2^-(i,j)]}_{x_2}$$
$$=b(i,j), \quad x_1^{\pm}, x_2^{\pm} \in \{0,+1\},$$

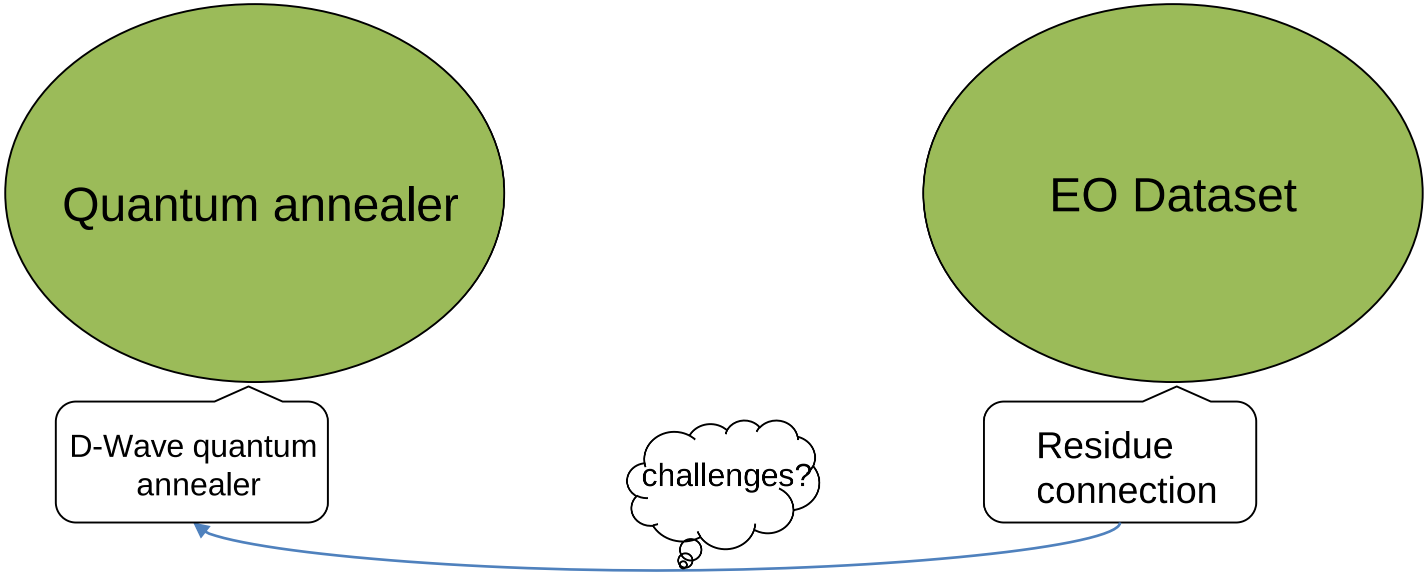
given residues $b(i,j) \in \{-1,0,+1\}$.

A QUBO formulation:

$$\min_{\{x_1^+,x_1^-,x_2^+,x_2^-\}} \Bigg[\sum_{i=0}^{N-2}\sum_{j=0}^{M-1} c_1(i,j)[x_1^+(i,j)+x_1^-(i,j)]$$
$$+\sum_{i=0}^{N-1}\sum_{j=0}^{M-2} c_2(i,j)[x_2^+(i,j)+x_2^-(i,j)]$$
$$+\lambda \sum_{(i,j)\in S} [x_1+x_2-b]^2\Bigg],$$

$$\sum_{(i,j)\in S} [x_1+x_2-b]^2 =$$
$$=\sum_{(i,j)\in S} 2x_1x_2-2b(x_1+x_2)+x_1+x_2.$$

What challenges may we encounter for using a D-Wave quantum annealer?



What challenges may we encounter for using a D-Wave quantum annealer?

- Embedding: logical variables to physical variables (Fig. 3)
  - Our QUBO variables: logical variables
  - D-Wave quantum annealer variables: physical variables

- Setting of annealing parameters:
  - Annealing time
  - A number of reads
  - The chain strength

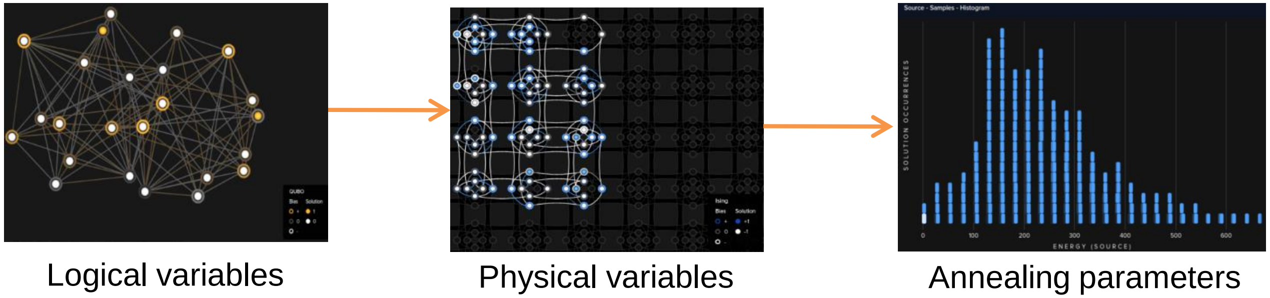
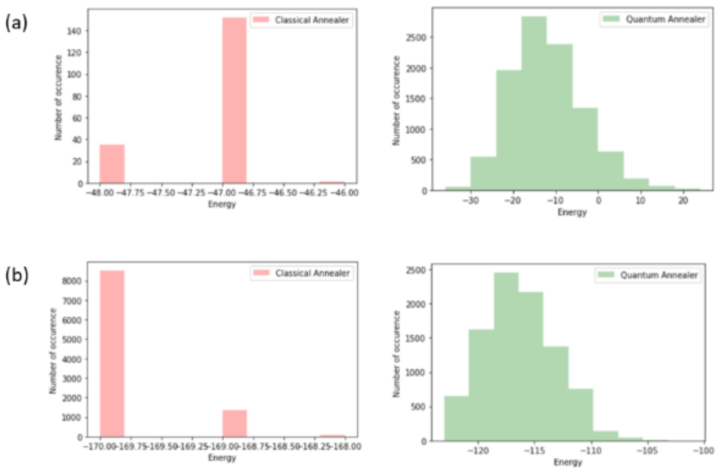


Fig. 3

A D-Wave quantum annealer for residue connection

- Embedding: logical variables to physical variables
  - minor-embedding offered by a D-Wave quantum annealer

- Setting of annealing parameters:
  1. Annealing time: $20 - 190\mu s$
  2. A number of reads: 1000
  3. The chain strength: 50

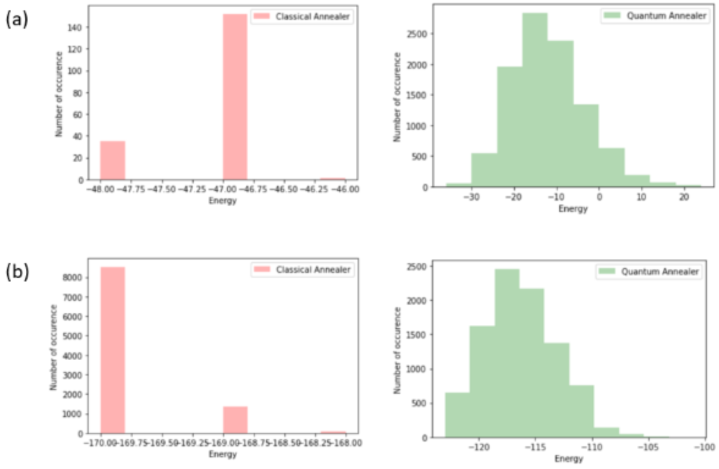


(a) 3×3 (b) 7×5

A D-Wave quantum annealer for residue connection

$$\min_{\{x_1^+, x_1^-, x_2^+, x_2^-\}} \left[\sum_{i=0}^{N-2} \sum_{j=0}^{M-1} c_1(i,j)[x_1^+(i,j) + x_1^-(i,j)] \right.$$
$$+ \sum_{i=0}^{N-1} \sum_{j=0}^{M-2} c_2(i,j)[x_2^+(i,j) + x_2^-(i,j)]$$
$$\left. + \lambda \sum_{(i,j)\in S} [x_1 + x_2 - b]^2 \right],$$

$$\sum_{(i,j)\in S} [x_1 + x_2 - b]^2 =$$
$$= \sum_{(i,j)\in S} 2x_1x_2 - 2b(x_1 + x_2) + x_1 + x_2.$$



(a) 3×3 (b) 7×5

Classical Annealer (CA) Vs Quantum Annealer (QA)

```python
#constraints of the residue connection as a quadratic term
for i in range(beq.shape[1]):#5
    for j in range(beq.shape[0]):#7

        if (results_dwave.first.sample['x1p(' + str(i)+','+str(j+1)+')']-
            results_dwave.first.sample['x1m(' + str(i)+','+str(j+1)+')']-
            results_dwave.first.sample['x1p(' + str(i)+','+str(j)+')']+
            results_dwave.first.sample['x1m(' + str(i)+','+str(j)+')']-
            results_dwave.first.sample['x2p(' + str(i+1)+','+str(j)+')']+
            results_dwave.first.sample['x2m(' + str(i+1)+','+str(j)+')']+
            results_dwave.first.sample['x2p(' + str(i)+','+str(j)+')']-
            results_dwave.first.sample['x2m(' + str(i)+','+str(j)+')'])==beq[j,i]:
            print("True")
        else:
            print("False")
```

QA	CA
True	True
True	True
False	False
True	True
False	False
True	False
False	False
True	True
True	True

NFM-to-QUBO for Integer variables

Our dataset is Cape Verde Volcano, and we consider residues such that $x_1^{\pm} \in$ Integers , $x_2^{\pm} \in$ Integers of a NFM formulation:

$$x_i^{\pm} = \sum_{k=0}^{n-1} 2^k q^*_{n\times i+k}\,, x_i^{\pm} \in Integers, q^*_{n\times i+k} \in \{0, +1\}.$$

$$\min_{\{x_1^+, x_1^-, x_2^+, x_2^-\}} \sum_{i=0}^{N-2} \sum_{j=0}^{M-1} c_1(i,j)[x_1^+(i,j) + x_1^-(i,j)]$$
$$+ \sum_{i=0}^{N-1} \sum_{j=0}^{M-2} c_2(i,j)[x_2^+(i,j) + x_2^-(i,j)],$$

Discussion

Earth observation datasets differ in their different physical and non-physical information such as their spatial dimensionality, their polarization states, and their spectral bands, due to the diverse sensors on different remote sensing platforms. Thus,

- Choice of a dataset plays a vital role in a quantum annealer (in general, quantum computers).
- A problem must be in a QUBO formulation.
- Connectivity of qubits and setting parameters in a quantum annealer affect solutions of a QUBO problem.
- Currently, no practical advantage of a quantum annealer is demonstrated (theoretically 10^8 faster than a classical annealer! by V. Denchev, S. Boixo, S. Isakov, et al., “What is the Computational Value of Finite-Range Tunneling?”, Phys. Rev. X, vol. 6, 031015, 2016.).

Conclusion

This work is a first attempt to enhance insights into a quantum annealer, and perceive some challenges to program the D-Wave quantum annealer for future Earth observation quantum technology. To leverage the D-Wave quantum annealer to obtain a better solution of the optimization problem, we chose a well-known optimization problem in Earth observation, and mapped it to a QUBO problem; in particular, the residue connection problem of the phase un-wrapping procedure in the InSAR.

Acknowledgement

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Thank you for listening!